Generalized Treatment of Optically-Induced Forces and Potentials in Optomechanically Variable Photonic Circuits

Peter T. Rakich and Miloš A. Popović

Research Laboratory of Electronics, Massachusetts Institute of Technology,
77 Massachusetts Ave, Cambridge, Massachusetts 02139
**e-mail: rakich@mit.edu*

Abstract: We establish a fundamental relationship between the phase and amplitude responses of an optomechanically variable photonic circuit and the forces and potentials produced by light. These results are illustrated through resonant and nonresonant multi-port systems.

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1. Introduction

Optical forces resulting from interacting modes and cavities can scale to large values as optical modes shrink to nanometer dimensions [2-3]. Such forces can be harnessed in fundamentally new ways when optical elements are free to mechanically adapt to them[1-6]. Through the creation of optically-induced potential-wells, we have illustrated that compelling new functionalities, such as self-aligning microcavities, can be realized [1]. Such forces and potentials may have a unique and useful role to play in functionalizing integrated photonic circuits, however, maximal exploitation of them requires the development of a clear and simple conceptual framework to guide design of such devices.

Currently, there are numerous means of understanding and computing optomechanical forces and potentials, most of which require specific knowledge of the device geometry [2-6]. In this paper, we reduce the problem of computing forces and potentials to its most essential form, illustrating that only the phase and amplitude response of an optomechanically variable device, and its dependence on the mechanical degree of freedom of interest, are necessary to compute the optomechanical forces and potentials that it will produce. We illustrate the tremendous simplification that this formalism yields when analyzing both resonant systems, such as optomechanically variable microcavities, and nonresonant optomechanically variable circuits of arbitrary form.

Through this analysis, we seek a general formulation of forces an potentials that naturally lends itself to the study of realistic experimental situations, most of which involve the interaction of an external light source, such as a laser, with a passive optomechanically variable device. By definition this is an "open-system", or one which explicitly couples to the outside world, requiring a formalism which is well suited for this situation. Throughout, we will represent light using the "photon" construct for the simplicity and insight that it provides in our formulation open-system analysis.

2. Simple case: lossless dispersive system

We begin by examining the energetics in the special case of a lossless system (seen in Fig 1(a)) with one input and one output, whose effect on the transmitted light is purely dispersive. The optical response of this device is assumed to be variable through motion of the generalized spatial coordinate,q. We derive the basic form for the optomechanical force and potential by examining the interaction of monochromatic light of frequency, ω, with this optomechanically variable system, showing that the potential energy of the system is intimately linked to the optical phase that the system induces on the transmitted wave(s) as the generalized spatial coordinate, q, is varied. For this special (lossless) case,

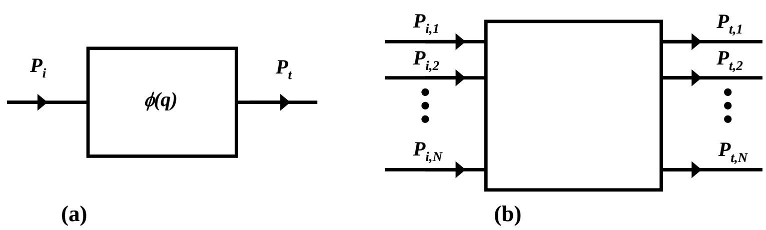

Fig. 1: **(a) Lossless optomechanically variable dispersive element with one input and output (b) Lossless optomechanically variable multi-port system with N inputs and outputs**

the incident wave simply experiences a coordinate dependent phase-shift, $\phi(q)$, in traversing the device, which we define as

$$exp[i(kz - \omega t)] \rightarrow exp[i(kz - \omega t + \phi(q))]. \tag{1}$$

In the classical limit (i.e. in the limit of large photon flux), one can express the powers entering and exiting the device as $P_i = \Phi_i \cdot \hbar\omega$ and $P_t = \Phi_t \cdot \hbar\omega'$ respectively. Here Φ and $\hbar$ are defined as the photon-flux and Plank's constant respectively, while, ω and ω' represent the incident and transmitted photon frequencies. Since this system is lossless, photon-flux must be conserved. For simplicity, we assume that the incident photon flux (Φ_i) is a constant given by Φ. In this case, conservation of photon-flux requires that $\Phi_i = \Phi_t = \Phi$. Thus, the only means by which electromagnetic energy (U_{EM}) can be modified is through a shift in photon frequency. Furthermore, since the phase response is an explicit function of the generalized coordinate, q, the frequency of the transmitted wave will only be altered when q is in motion. To examine the energetics of this system, we assume that q takes on explicit time dependence,yielding a time-dependent phase of the form, $\phi(q(t))$. The instantaneous frequency shift on the transmitted wave, resulting from a time-dependent phase-shift, $\phi(t)$, will be $\delta\omega = -\dot{\phi}$. Therefore the frequency of photons exiting the device, ω', can be express as $\omega' = (\omega + \delta\omega) = (\omega - \dot{\phi})$, allowing us to write the incident (P_i) and transmitted (P_t) powers as

$$\begin{aligned} P_i &= \Phi \cdot \hbar\omega & (2) \\ P_t &= \Phi \cdot \hbar\omega' & (3) \\ &= \Phi \cdot \hbar(\omega - \dot{\phi}). & (4) \end{aligned}$$

From the above, we see that a time-dependent evolution of the phase must correspond to a time-dependent change of electromagnetic field energy, according to

$$\frac{dU_{EM}}{dt} = (P_t - P_i) = -\Phi \cdot \hbar\dot{\phi}. \tag{5}$$

Since $\dot{\phi}$ can be expressed as $\frac{d\phi}{dt} = \frac{d\phi}{dq}\frac{dq}{dt}$, a general time independent form of equation (6) can be found, yielding

$$\frac{dU_{EM}}{dq} = -\Phi \cdot \hbar\frac{d\phi}{dq}. \tag{6}$$

Though the total energy of an "open-system" is difficult to define, integration of the above expression reveals that the change in electromagnetic energy induced through motion of the coordinate, q, can be expressed as

$$\Delta U_{EM}(q) = -\Phi \cdot \hbar \int_{q_o}^{q} \frac{d\phi}{dq'} \cdot dq' = -\Phi \cdot \hbar\phi(q) + \alpha. \tag{7}$$

Here, q_o is an arbitrary point of origin. Since the change in energy of the system corresponds to mechanical work performed through motion the generalized coordinate, q, $\Delta U_{EM}(q)$ can be interpreted as the optomechanical potential $U_{eff}(q)$ of the system. Dropping the superfluous constant term, α, the potential is simply

$$U_{eff}(q) = -\Phi \cdot \hbar\phi(q). \tag{8}$$

Thus, in the special case of a purely dispersive system, we see that the optomechanical potential is given by the phase-change imparted on the transmitted wave as the generalized coordinate varies. It follows that the above optomechanical potential is conservative (provided that the trajectory of the generalized coordinate, q, is single valued). It is important to note that this analysis assumes adiabatic (or quasi-static) evolution of the coordinate, q, such that $|\dot{\phi}| << \frac{1}{\tau_p}$, where τ_p is the photon-lifetime. This is equivalent to requiring that the system is (to a good approximation) at steady-state with the input signal as the coordinate evolves. Finally, we remark that forces generated by light in the direction of the coordinate, q, are given by $F_q = -dU_{eff}/dq$, and are found to be identical to those computed through closed-system analysis of the equivalent physical systems[1-3].

3. Generalization to multi-port systems

Thus far, we have illustrated how the "open-system" optomechanical potential can be computed using the photon-picture, in the special case of a lossless dispersive optomechanically variable device. Next, we generalize this "open-system" formulation of the optomechanical potential using the photon-picture, showing that a simple integral form of the optomechanical potential can always be written if the amplitude and phase-response of a multi-port optomechanical system (as a function of the generalized coordinate of interest) are known.

In general, the *phase* and *amplitude* response will change as the generalized coordinate is varied in a multi-port system. Therefore, we can no longer expect that the phase-response will possess complete information for the optomechanical potential. For simplicity, the multi-port system is schematically illustrated in Fig. 1(b), showing spatially separated input and output ports to the device (however, in reality they needn't be). We assume that N input signals, of fixed phase and amplitude, enter the multi-port system from the left with powers specified by $P_{i,k} = \Phi_{i,k} \cdot \hbar\omega$. This implies that the incident photon fluxes, $\Phi_{i,k}$, must be fixed, however, motion of the optomechanical degree of freedom will, in general, effect the transmitted photon fluxes, $\Phi_{t,k}(q)$, and frequencies, ω'_k, meaning that the transmitted power of the k^{th} output-port can be expressed as $P_{t,k}(q) = \Phi_{t,k}(q) \cdot \hbar\omega'_k$. We again assume that the system is lossless, requiring conservation of photon flux, meaning that $\sum_k \Phi_{i,k} = \sum_k \Phi_{t,k}(q) = \Phi_{tot}$ must hold for all values of q. Assuming a time-dependent evolution of q, as before, the incident (P_i) and transmitted (P_t) powers can be expressed as

$$P_i = \hbar \cdot \sum_k \Phi_{i,k} \cdot \omega = \Phi_{tot} \cdot \hbar\omega \tag{9}$$

$$P_t = \hbar \cdot \sum_k \Phi_{t,k}(t) \cdot \omega'_k \tag{10}$$

$$= \Phi_{tot} \cdot \hbar\omega - \hbar \cdot \sum_k \Phi_{t,k}(t) \cdot \dot{\phi}_{t,k}(t). \tag{11}$$

Above, we have used the fact that $\omega'_k = (\omega - \dot{\phi}_{t,k}(t))$. Power conservation tells us that the time derivative in the electromagnetic energy (U_{EM}) is given by $(P_t - P_i)$. Removing the time dependence from this relationship, and replacing U_{EM} with U_{eff} we have

$$\frac{dU_{eff}}{dq} = -\hbar \cdot \sum_k \Phi_{t,k}(q) \cdot \frac{d\phi_{t,k}}{dq}. \tag{12}$$

Note that the quantity $-dU_{eff}/dq$ is equivalent to the force produced in the direction of the generalized coordinate. Through integration of the above relation, we find the following general form for the optomechanical potential

$$U_{eff}(q) = -\hbar \cdot \int \left[\sum_k \Phi_{t,k}(q) \cdot \frac{d\phi_{t,k}}{dq} \right] \cdot dq. \tag{13}$$

The above is a remarkably general form of the optomechanical potential for a lossless optomechanically variable circuit with N inputs and N ouputs. Apparently, no knowledge of the internal workings of this system is necessary in order to compute the optomechanical force and potential that it will create. For fixed input conditions, we need only know the amplitude and phase of the transmitted waves as the generalized optomechanical coordinate, q, is varied.

4. conclusions

Through "open-system" analysis using the "photon-picture", the problem of computing forces and potentials has been reduced to its most essential form, illustrating that only the phase and amplitude response of an optomechanically variable device, and its dependance on the mechanical degree of freedom of interest are necessary to compute the optomechanical forces and potentials that it will produce. We have shown that, in the case of a lossless multi-port system with a single mechanical degree of freedom, the optomechanical potential can be written in a very general and simple integral form. This represents an important first step towards the use of optomechanical systems to solve general problems in integrated photonics, allowing scientists and engineers to examine new optomechanical functionalities in a fundamentally new way. Rather than examining the force and potential that a specific device yields, one can ask whether a certain optomechanical potential is physically realizable, and if so, what constraints must be satisfied.

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