Spectral anomalies due to coupling-induced frequency shifts in dielectric coupled-resonator filters

Christina Manolatou[†], Miloš A. Popović[‡], Peter T. Rakich, Tymon Barwicz, Hermann A. Haus and Erich P. Ippen
Research Laboratory of Electronics, Massachusetts Institute of Technology, 77 Massachusetts Ave, Cambridge, Massachusetts 02139
[†]krist@alum.mit.edu, [‡]milos@mit.edu

Abstract: Coupling-induced frequency shift is identified as a source of spectral response degradation in higher-order coupled-resonator add/drop filters that must be compensated in design. The theoretical basis and experimental verification are presented, and generic solutions proposed.

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1. Introduction

Coupled dielectric resonator filters are candidates for integrated WDM channel add/drop applications, with well-developed synthesis techniques [1-3]. In response synthesis, precise control of the resonance frequencies and mutual couplings is required. In series-coupled add/drop filters, for example, the uncoupled resonance frequencies must be identical [1]. Herein, a coupling-induced frequency shift (CIFS) in resonators is identified as an effect that may hinder the realization of the desired response if left uncompensated. The frequency shift can be explained as the self-excitation of a resonant mode, via the index perturbation of a second structure, with a temporal phase shift such that its frequency is modified. The shift may be negative or positive depending on the dominant of two contributing effects to be described. In what follows, the CIFS is theoretically explained within coupled mode theory formalism, and corroborated by FDTD simulations. Degradation of higher-order filter responses due to the CIFS is investigated by way of a third-order series-coupled microring filter model, with comparison to experimental results from such a filter. A modification of filter synthesis is suggested to recover ideal responses. Generic solutions for compensation of the relevant resonators are proposed.

2. Theoretical basis for negative and positive coupling-induced frequency shifts

Coupling of modes in time (CMT) [4] provides a treatment of the interaction of arbitrary coupled dielectric resonators. Evolution of the amplitudes $\vec{a}(t)$ of uncoupled modes in the coupled system can be described by

$$\frac{d}{dt}\vec{a} = j\boldsymbol{\omega}\cdot\vec{a} - j\mathbf{W}^{-1}\cdot\mathbf{M}\cdot\vec{a} = j\boldsymbol{\omega}\cdot\vec{a} - j\boldsymbol{\mu}\cdot\vec{a} \qquad (1)$$

where $\boldsymbol{\omega}$ is a diagonal matrix of uncoupled resonance frequencies, $\mathbf{W}$ is the energy non-orthogonality matrix, and $\mathbf{M}$ is a coupling overlap matrix. $\boldsymbol{\mu}$ represents a total effective coupling matrix with respect to mode amplitudes taking energy non-orthogonality into account. The coupling-induced frequency shift (CIFS) results from self-coupling terms on the diagonal of $\boldsymbol{\mu}$. For the resonator associated with amplitude a_1 of a total of two modes, the shift is

$$\delta\omega_1 = -\mu_{11} = -\frac{M_{11} - \frac{W_{12}}{W_{22}}M_{21}}{W_{11} - \frac{W_{12}}{W_{22}}W_{21}}. \qquad (2)$$

In absence of coupling $\delta\omega_1 = 0$, and resonators oscillate at natural frequencies in $\boldsymbol{\omega}$. Under coupling, the CIFS in (2) is generally non-zero. It can be critical in filter design and can seriously degrade performance if left uncompensated.

We briefly consider the physical origin of the CIFS. For orthogonal modes (e.g. a resonator perturbed by a nearby dielectric object with no relevant modes of its own) $W_{12} = W_{21} = 0$, and from (2) the frequency shift is negative since M_{ii}/W_{ii} is positive definite in the lossless case. This is an intuitive result if one considers the wave equation, or its stationary integral for frequency: a positive refractive index perturbation is introduced, so the frequency must decrease. In the case of adjacent evanescently-coupled resonators (e.g. [1,2]), the basis is normally not orthogonal. A second, positive CIFS contribution then arises due to the shared cross-energy of the modes. Since cross-coupling M_{21} (source of power exchange) is a large term relative to self-coupling M_{11}, the net CIFS could be found positive. This initially unintuitive result is easily understood in the spatial picture that follows.

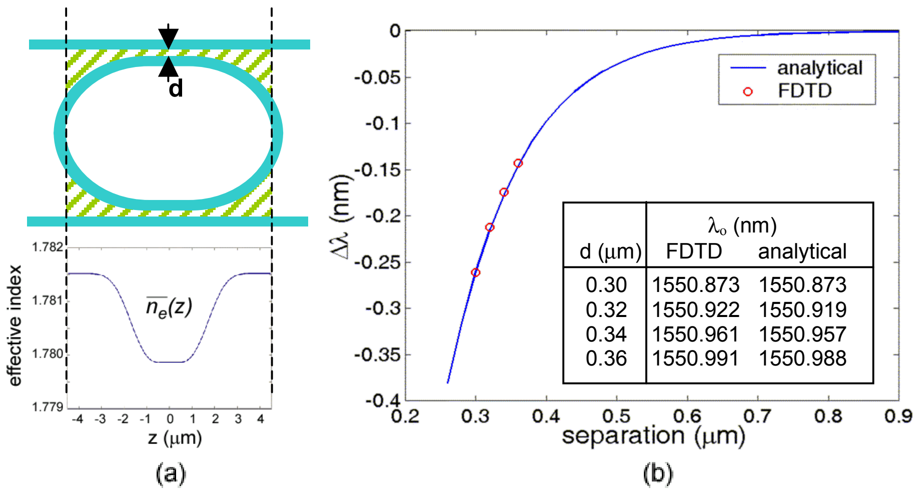

Fig. 1. Coupled-mode theory prediction and FDTD simulation of coupling-induced frequency shift. (a) geometry of CMT integration of the self-phase shift and effective index vs. position; (b) CIFS from coupled mode theory and FDTD.

Coupling of modes in time provides a framework for numerical evaluation of CIFS in standing or traveling wave resonators of arbitrary shapes. The shift may also be obtained using a resonant-mode solver or FDTD.

3. Coupling of modes-in-space picture for microrings and racetracks and FDTD verification of the CIFS

For traveling wave resonators including rings and racetracks, the CIFS is simply understood by using a coupling of modes in space [4] picture to consider the self-phase-shift accumulated in directional couplers. The propagation constant β_i of a waveguide is modified by the diagonal terms $\delta\beta_i$ of a coupling matrix analogous to that in (1). To translate the accumulated phase shift into a frequency shift for a ring/racetrack resonator, an approximate formula may be used that integrates the perturbation-induced $\delta\beta_i$ along the couplers to find a total phase shift and CIFS:

$$\delta\beta_1 = \frac{K_{11} - \frac{P_{12}}{P_{22}} K_{21}}{P_{11} - \frac{P_{12}}{P_{22}} P_{21}}; \qquad \delta\omega_1 = -\frac{\int_{L_c} \delta\beta_1(\omega_o, z)\,dz}{\frac{L - L_c}{v_g} + \int_{L_c} \frac{dz}{v_{g,c}(z)}} \qquad \text{(3a,b)}$$

where $\delta\beta_1$ is given for mode 1 of two modes, **K** is the coupling overlap matrix and **P** is the non-orthogonality matrix [4]. For (3b), L is total cavity length, L_c is the length of coupler regions, $\delta\beta_1(\omega_o, z)$ is a spatially varying $\delta\beta_1$ from (3a) at the free-running frequency ω_o, and v_g and $v_{g,c}(z)$ are mode group velocities in uncoupled and coupler regions.

This picture confirms that the CIFS can be of either sign. K_{11} is the usual positive effective index (negative CIFS) contribution due to the presence of the high index adjacent bus waveguide or resonator. The second term in (3) that gives a positive CIFS contribution can be understood by considering two weakly guided TE coupled slabs of half the width necessary to cut off the second guided mode. At zero wall-to-wall spacing (strong coupling regime), the two guides merge and the antisymmetric mode becomes cut off, while the symmetric morphs into the fundamental guided mode. In approaching this situation, clearly the antisymmetric mode's effective index drops much faster than that of the symmetric mode rises. Thus there is a negative contribution to the average effective index of the two supermodes, which for identical coupled waveguides corresponds to a $\delta\beta < 0$ in (3), or a CIFS > 0.

A 2D FDTD simulation of a single TE racetrack resonator (L = 34.6μm) add/drop filter in which the anomalous positive CIFS was first observed is shown in Figure 1a. The free-running frequency was found as 1551.134nm, but under coupling to two bus waveguides it shifted to values given in the table in Fig. 1b, resulting in a positive CIFS of over -0.25nm or +30GHz. Figure 1b shows the results of a coupling-of-modes prediction of the frequency shift, using (3a) and (3b) integrated over the couplers (shaded region in Fig. 1a) of varying waveguide spacing. The close match of the exact CIFS and the coupling of modes prediction supports the given explanation of the effect.

4. CIFS impact on higher-order filter spectra and proposed correction to filter synthesis and design

While the impact on a single-resonator filter is a simple frequency shift in the response spectrum, the impact of the CIFS on higher-order filters is more serious. In synthesizing a prototype filter response, a set of desired resonant frequencies (identical for series-coupled add-drop filters) and normally different ring-ring and ring-bus couplings is

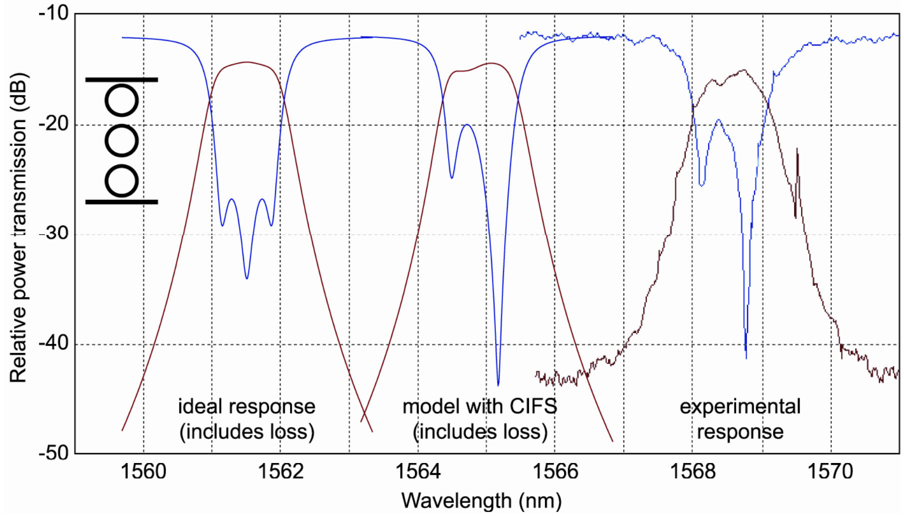

Fig. 2. Drop and thru port frequency responses of a 3rd-order filter using identical microrings (inset diagram): ideal theoretical response (left plot); theoretical model incorporating the expected net CIFS in the middle ring of +20GHz from CMT (middle plot); experimental measurement of the response of fabricated filter based on three identical rings (right plot). Loss is fit into the models to match experiment. Theoretical spectra are displaced horizontally for clarity in the plot.

to be achieved [1-3]. The CIFS due to different coupler gaps will be different, and will cause resonators to have different effective resonant frequencies within the same filter. In practical add/drop filters of [1], resonator-bus coupling is stronger than inter-resonator coupling, so the most significant frequency shift is normally seen to be due to the bus waveguide. Thus, the first and last resonator in the 'series' can be expected to suffer from the largest shift.

The effect of non-degenerate resonant frequencies is shown for a 100GHz-wide third-order Chebyshev (0.2dB ripple) add/drop filter model in Fig. 2 (left and center). The plot shows an ideal response with ring loss included, but no CIFS (left); and the same filter with a +20GHz CIFS in the middle microring (middle plot). A significant degradation of the thru-port rejection is seen. In addition, the drop spectrum acquires a slight asymmetry and ripple.

On the right in Fig. 2 is the measured spectral response of a third-order add/drop filter fabricated in SiN by electron-beam lithography [5]. Based on the dimensions and materials of this filter, an expected CIFS of {-24GHz, -2GHz, -24GHz} was computed for the three rings, respectively using a vectorial CMT. This corresponds to a +22 GHz net shift for the middle ring, used above in the model for comparison. The loss in the model above (Q = 12,000) was chosen to match the experimental insertion loss, though the ring Q's were assumed equal. This assumption makes for some discrepancy as the middle ring is expected to have lower loss than its more strongly coupled neighbours, but the experimental spectrum confirms a frequency difference among microrings of ~20GHz. While this difference could be caused by slight size differences between the resonators due to fabrication, a measurement of dimensions in [5] suggests that the primary contribution to the spectrum asymmetry is due to the CIFS. Neither an asymmetric coupling nor ring loss distribution can account for an asymmetric spectrum.

Naturally, the solution is to design each resonator such that its effective frequency including CIFS in the final configuration is the design frequency. In practice, this means pre-distorting the isolated resonator frequency by the negative of the CIFS computed, by slightly redesigning each resonator to alter the frequency but maintain coupling.

In general, CIFS pre-distortion is accomplished by slightly changing the size, shape or refractive index of a resonator, or by loading it with additional dielectric structures. In the case of microrings and racetracks, this can be accomplished most simply by a change in the radius (i.e. effective path length), waveguide width or core index. Thus, the final design of a series-coupled microring add/drop filter as in [1] calls for slightly non-identical resonators with non-degenerate resonance frequencies prior to coupling.

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