A Discrete Resonance, All-Order Dispersion Engineering Method for Microcavity Design for Four-wave Mixing

Cale M. Gentry[1*], Xiaoge Zeng[2], and Miloš A. Popović[1†]

[1] *Department of Electrical, Computer, and Energy Engineering, University of Colorado Boulder, CO, 80309, USA*
[2] *Department of Physics, University of Colorado Boulder, Colorado, 80309, USA*
[*] *cale.gentry@colorado.edu,* [†] *milos.popovic@colorado.edu*

Abstract: We propose a rigorous method for tailoring the dispersion of azimuthally-symmetric microresonators for four-wave mixing applications and show example designs. The method implicitly includes momentum conservation and directly reveals phase mismatch via resonance detuning, avoiding Taylor expansions.

OCIS codes: (130.3990) Micro-optical devices, (130.2035) Dispersion compensation devices, (190.4380) Four-wave mixing

Four-wave mixing (FWM) in microresonators is the subject of current research efforts with applications including wavelength conversion [1], correlated photon-pair sources [2], and optical frequency comb generation [3]. A critical requirement for efficient FWM is the conservation of energy and momentum between pump and generated signal-idler photons (phase-matching in time and space). A common approach is to utilize modes of different longitudinal order separated by multiple free-spectral ranges (FSRs) of a cavity for the pump, signal, and idler modes. The field/Purcell enhancement at these resonances greatly improves FWM efficiency compared to that of a waveguide [4]. While the resonant modes are intrinsically momentum matched [1], dispersion can lead to unequal (in frequency) FSRs [Fig. 2(b)] violating the energy matching condition and results in at least one of the three photons being off-resonance, diminishing the potential FWM efficiency of the device. A common approach to engineering the dispersion in resonators comes from the same method in waveguides and involves finding a resonator structure with a zero dispersion point near the intended pump wavelength [1, 5] as shown in Fig. 1(a). The zero dispersion point arises from where the group velocity dispersion (GVD), proportional to the second derivative of the propagation constant with respect to angular frequency $d^2\beta/d\omega^2$, vanishes. This method relies on a 2nd-order approximation and does not guarantee phase-matching between resonant modes and often (especially in the case of combs and widely separated signal and idler) results in the need to compute higher order dispersion terms [6]. More importantly, setting all orders of dispersion to zero is an over-constraint since dispersion need not be zero at all wavelengths around the pump, and in principle one could achieve perfect resonance matching without having any one single dispersion order equal to zero.

In this paper, we propose and demonstrate direct computation of the energy matching between resonances of a microresonator. Since waveguides inherently support a continuum of wavelengths, it is natural to assume energy matching ($2\omega_p = \omega_i + \omega_s$) and to simulate the effect of material and modal dispersion on momentum matching. The phase mismatch between wavelengths is often found through Taylor expansion about the pump frequency to be $\Delta\beta = 2\beta_p - \beta_s - \beta_i = \sum_{n=2,4,6...}(2/n!)\beta_n(\omega_s - \omega_p)^n$ where β_n is the n^{th} derivative of the propagation constant β with respect to angular frequency ω at the pump. It is readily apparent that reducing β_2 to zero only approximates

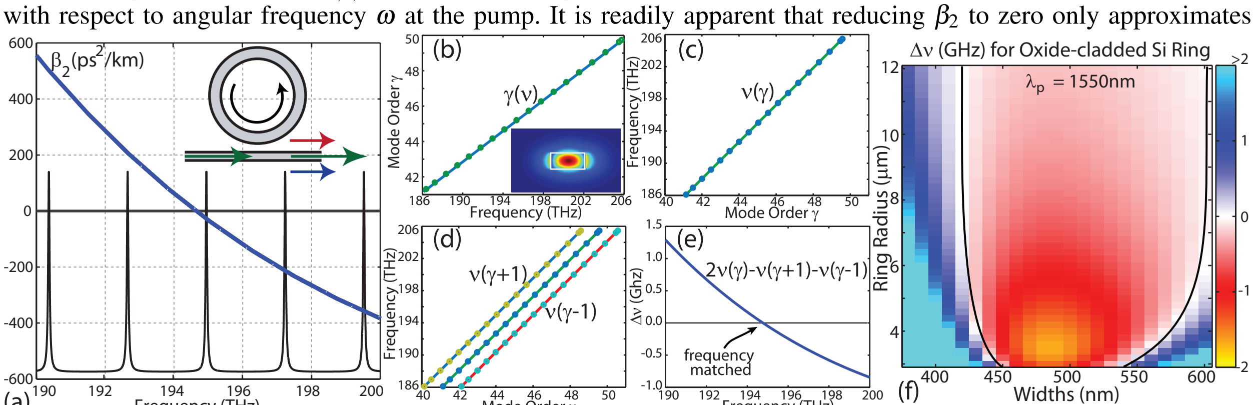

Fig. 1: (a) Conventional method of dispersion engineering (Find zero β_2). Our proposed method of simulating the azimuthal mode order γ vs frequency ν (b), performing a coordinate rotation to get ν versus γ, and finding the difference between the frequencies at other FSRs (d) to determine the frequency mismatch (e) between resonances caused by dispersion. (f) An example frequency mismatch simulation of a 220 nm tall silicon ring resonator with swept radius and width parameters.

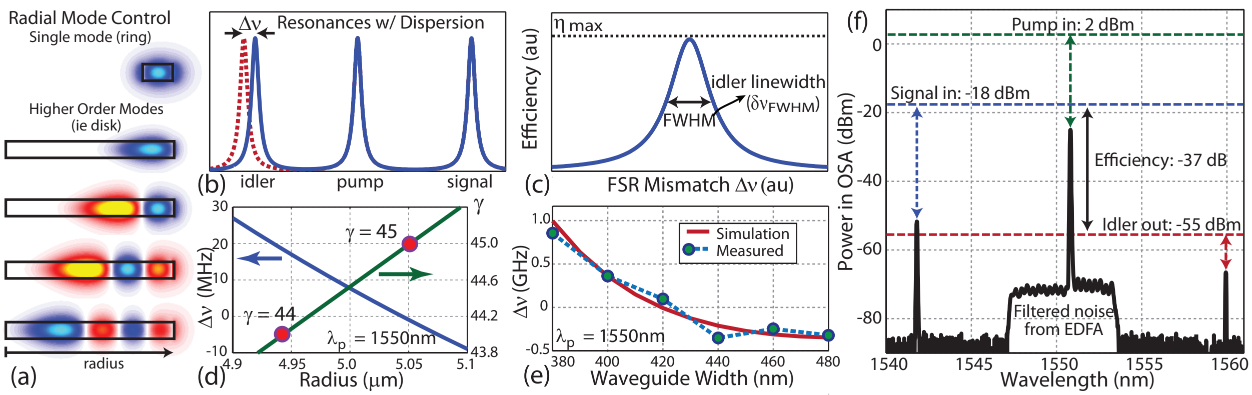

Fig. 2: (a) Radial mode order provides another degree of freedom for dispersion engineering that our method can readily include. (b) Frequency mismatch between dispersive cavity (solid) and an ideal dispersionless cavity with equal FSRs (dashed). (c) Frequency mismatch between modes leads to a reduction in a Lorentzian shaped efficiency profile with FSR mismatch with the same linewidth as the idler resonance. (d) Example simulation showing that despite allowing non-integer γ terms in our simulations a slight change of radius can achieve the resonance condition with negligible effect on the performance. (e) Measured and simulated FSR mismatch for oxide cladded silicon rings with 10 μm outer radius. (f) OSA spectrum of seeded four-wave mixing in designed cavity.

phase-matching and becomes increasingly inaccurate at large separation between pump and signal-idler frequencies. Resonators, on the other hand, intrinsically guarantee momentum matching between resonances making it more intuitive to assume momentum matching and simulate the effect of dispersion on the frequency spacing between resonant modes, which can be performed without expansion approximations. We achieve this by simulating a resonant structure in an azimuthal mode order (γ) mode solver for various input frequencies while including the material dispersion found in [7]. This provides a monotonic function (assuming positive group velocity) of $\gamma(\nu)$ [Fig. 1(b)] from which we can obtain $\nu(\gamma)$ by inverting the function (i.e. switching ordinate and abscissa) [Fig. 1(c)]. From here, we define $\nu(\gamma)$ as the pump mode, $\nu(\gamma+N)$ as the signal mode, and $\nu(\gamma-N)$ as the idler mode where N is the number of FSRs separating the pump and signal-idler [$N = 1$ case is shown in Fig. 1(d)]. The frequency mismatch between the discrete resonances can then be found by $\Delta\nu = 2\nu(\gamma) - \nu(\gamma+N) - \nu(\gamma-N)$ and mapped back to the pump frequency axis as in Fig. 1(e). The process can then be repeated for various structural degrees of freedom (widths, heights, materials etc) over preferred wavelength ranges. The method also may be modified for additional degrees of freedom such as radial mode order in disk-like microcavities [Fig. 2(a)]. We provide an example simulation of an oxide-cladded silicon ring resonator in Fig. 1(f) with varying radii and widths.

While the method allows for non-integer values of azimuthal mode order γ which by definition do not correspond to resonant modes, we note that a slight change in radius can shift them to integer values with negligible effect to the phase-matching properties. Fig. 2(d) shows as an example that for an arbitrarily selected cavity a maximum shift of $\sim$50 nm in radius is required for the resonance condition, resulting in a 10 MHz shift in FSR mismatch. Since the dependence of efficiency of a FWM source is Lorentzian with respect to frequency mismatch [Fig. 2(c)] (with a linewidth equal to that of the idler resonance's [4]), this shift is negligible for resonators with modestly high quality factors ($Q < 10^6$). In general absolute resonance location is much more sensitive to fabrication than dispersion. The proposed method was then utilized to design a dispersion engineered oxide-cladded 220 nm thick silicon ring with three phase-matched modes centered near 1550 nm. Comparison of measured FSR mismatch (characterized using the nearest resonance to 1550 nm as the pump) of the 420 nm wide, 10 μm radius ring to the simulation is shown in Fig. 2(e) with generated FWM spectrum in Fig. 2(f).

In summary, we have proposed a rigorous method of engineering all-order dispersion in microcavities for four-wave mixing across discrete resonant modes. We also demonstrated initial use of the method in the design of a single ring four-wave mixing device. Further applications of this method may include widely-detuned photon-pair sources and mode-locked ultra-broadband Kerr frequency combs. We note that this method may be similarly applied to designing devices based off other nonlinear optical processes by appropriately re-defining $\Delta\nu$.

This work was supported by a 2012 Packard Fellowship for Science and Engineering (Grant #2012-38222).

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