Wavelength Conversion in Modulated Dual-Mode Resonators and its Equivalence to a Linear Filter Model

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Abstract: We present a resonant modulation scheme that enables efficient wavelength conversion. The system maps onto linear filter equations that provide straightforward analysis and optimized design. Efficiencies of silicon carrier-plasma modulator implementations are estimated.

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Recent work has utilized the resonantly enhanced sidebands resulting from modulation of a microring's resonant frequency for RF signal generation [1]. This approach has the limitation that large optical cavities must be used which limits the achievable modulation efficiency due to large capacitance. In this paper, we propose a coupled cavity, dual-mode resonator modulator system as an efficient approach to wavelength conversion and RF signal generation. The approach allows maximum conversion efficiency for a given loss Q and strength of modulation. We present a coupling of modes in time (CMT) model for modulated coupled resonators and show its equivalence to simple, linear coupled-resonator filters. This approach allows well-known linear filter designs to be directly mapped to modulated coupled resonators and utilized to design the structure for optimum performance.

The CMT equations for two coupled, lossless resonators with resonant frequencies modulated anti-symmetrically are given by

$$\frac{d}{dt}\begin{pmatrix}a_1\\a_2\end{pmatrix} = j\begin{pmatrix}\omega_o + \frac{\delta\omega_m}{2}\cos(\omega_m t) & 0\\ 0 & \omega_o - \frac{\delta\omega_m}{2}\cos(\omega_m t)\end{pmatrix}\cdot\begin{pmatrix}a_1\\a_2\end{pmatrix} - j\begin{pmatrix}0 & \mu\\ \mu & 0\end{pmatrix}\cdot\begin{pmatrix}a_1\\a_2\end{pmatrix} \quad (1)$$

where a_1, a_2 are the energy amplitudes in resonators 1 and 2, ω_o is the uncoupled resonant frequency of resonators 1 and 2, $\delta\omega_m$ is the amplitude of modulation, ω_m is the modulation rate, and μ is the coupling between resonators. By solving for the unmodulated ($\delta\omega_m = 0$) supermodes of Eqn. 1 and rewriting the CMT equations in terms of the supermode amplitudes, we get

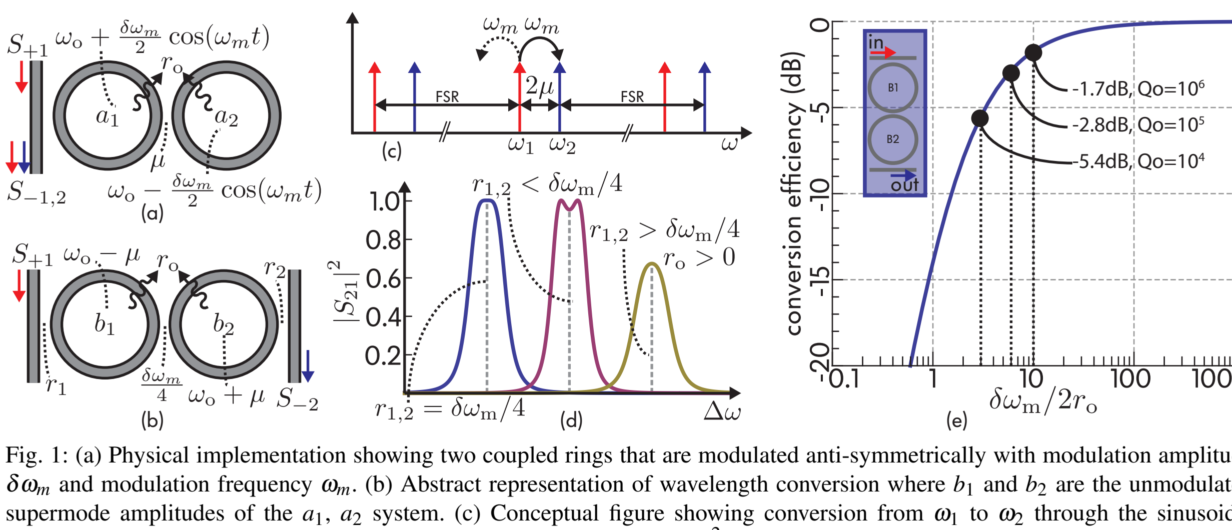

Fig. 1: (a) Physical implementation showing two coupled rings that are modulated anti-symmetrically with modulation amplitude $\delta\omega_m$ and modulation frequency ω_m. (b) Abstract representation of wavelength conversion where b_1 and b_2 are the unmodulated supermode amplitudes of the a_1, a_2 system. (c) Conceptual figure showing conversion from ω_1 to ω_2 through the sinusoidal modulation. There is no coupling between adjacent-order FSRs. (d) $|S_{21}|^2$ for the system in (b); maximally flat (left), Chebyshev (middle), and optimal design with loss (right). (e) Wavelength conversion efficiency as a function of modulation strength normalized to intrinsic bandwidth. The optimal choice of decay rates is used to maximize conversion efficiency.

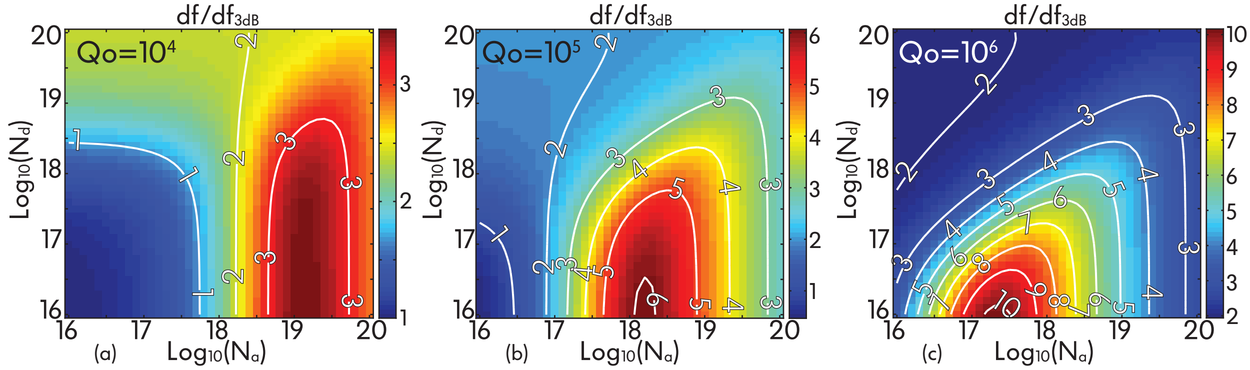

Fig. 2: Ratio of resonance frequency shift to intrinsic linewidth in silicon carrier plasma-dispersion effect modulators for three radiation limited quality factors versus accepter/donor implant concentrations. (a) $Q_o = 10^4$. (b) $Q_o = 10^5$. (c) $Q_o = 10^6$.

$$\frac{d}{dt}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = j\begin{pmatrix} \omega_o - \mu & 0 \\ 0 & \omega_o + \mu \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} + j\begin{pmatrix} 0 & \frac{\delta\omega_m}{2}\cos(\omega_m t) \\ \frac{\delta\omega_m}{2}\cos(\omega_m t) & 0 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad (2)$$

By substituting $b_1 = B_1 e^{j(\omega_o - \mu)t}$ and $b_2 = B_2 e^{j(\omega_o + \mu)t}$, we arrive at the evolution equations for the envelope amplitudes. Expanding the cosine term into its exponential definition leads to two exponential terms with arguments $\propto (\omega_m \pm 2\mu)t$. Since the envelope amplitudes are at 0 frequency, we keep only the terms that can achieve phase matching, $e^{\pm j(\omega_m - 2\mu)t}$. The envelope amplitude equations are now

$$\frac{d}{dt}\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = j\frac{\delta\omega_m}{4}\begin{pmatrix} 0 & e^{-j(\omega_m - 2\mu)t} \\ e^{j(\omega_m - 2\mu)t} & 0 \end{pmatrix} \cdot \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \quad (3)$$

By setting $\omega_m = 2\mu$ (i.e. the modulation rate equal to the frequency splitting between the supermodes), the equations reduce to the same equations as derived for the envelope amplitudes of a static, 2-ring coupled resonator add-drop filter [2]. Next, ports are added to the system and we can apply steady-state CMT analysis and solve for the conversion efficiency from port 1 (at frequency $\omega_o - \mu$) to port 2 (at frequency $\omega_o + \mu$). This results in an equation for $|S_{21}|^2$ which is plotted in Fig. 1(d) for different relationships between the decay rates and the modulation strength. Standard filter synthesis for coupled resonators [2] is used to design the relationship between the ring-bus decay rates, $r_{1,2}$, and the modulation strength, $\delta\omega_m$. The input and output ports have envelope amplitudes $\vec{S}_+$ and $\vec{S}_-$, respectively. It is important to note that $\Delta\omega$ is both the detuning of the input wave, S_{1+}, from the center frequency of b_1 and the detuning of the output wave, S_{2-}, from the center frequency of b_2. We plot the normalized conversion efficiency in Fig. 1(e).

To estimate the conversion efficiency in realistic systems, we consider a symmetric p-n junction carrier plasma-dispersion effect modulator that is widely used in silicon photonics [3]. With this type of modulator, there is a tradeoff between the achievable resonance shift and the broadening of the intrinsic linewidth due to the losses introduced by the implants. Using experimental fits for $\Delta n/\Delta N_{A,D}$ and $\Delta\alpha/\Delta N_{A,D}$ from [4], curves for the achievable resonance shift normalized to the intrinsic linewidth are generated (Fig. 2). The combination of Figs. 1(e) and 2 gives the estimation of conversion efficiency for plasma-dispersion effect resonant modulators in silicon. The maximum normalized frequency shift ranges from $\mathrm{df}/\mathrm{df}_{3dB} = 3$ to 10 in Fig. 2. This maps directly to the x-axis in Fig. 1(e) and corresponds to conversion efficiencies between -5.4 dB to -1.7 dB. It becomes increasingly difficult to electrically deplete the cavity when using large implant concentrations, so the electrical design must be considered jointly with Fig. 2.

A modulated coupled-resonator system allows controlled coupling of only two wavelengths, permits independent design of wavelength shift (coupling-induced resonance splitting) and the modulation mechanism. We have shown that this class of time-dependent systems maps to linear filters, and leveraging these techniques can bring considerable sophistication to the design of modulator-based photonic systems, especially with the recent progress in complex on-chip electronic photonic systems.

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