# Analysis of Leaky-Wave Microphotonic Structures with a Complex-Wavevector Photonic Band Structure Solver

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**Abstract:** A finite-difference, complex-wavevector photonic band structure solver with perfectly matched layer absorbing boundaries is presented. Modal properties of leaky-wave structures, such as silicon photonic grating couplers and waveguide crossing arrays, are shown.

**OCIS codes:** (050.1755) Computational electromagnetic methods; (130.3120) Integrated optics devices.

## 1. Introduction

Periodic structures, such as fiber-to-chip grating couplers [1,2], waveguide crossing arrays [3,4], and photonic crystal cavities and waveguides [5], play an important role in the design of integrated photonic devices and circuits. A natural way to design these periodic structures is through photonic band structure analysis with numerical band solvers being readily used [6]. Traditional band solvers typically have an input of real wavevector, $k$, and output of real angular frequency, $\omega$, or complex $\omega$ for radiating structures, and, as a result, do not readily provide band structure information within band gaps where the wavevector is complex. For design of periodic structures that make use of complex-wavevector modes for continuous-wave excitation with a real target wavelength, including adiabatically tapered band structure devices like photonic crystal nanobeam cavities [5], a complex wavevector solver is desired [7,8]. Additionally, many periodic structures support leaky waves, such as grating couplers with deliberately engineered radiation and crossing arrays with undesirable, parasitic radiation. These structures require Perfectly Matched Layer (PML) absorbing boundary conditions at computational domain boundaries to prevent fictitious reflection in the solution space [9].

In this paper, we demonstrate a numerical finite-difference frequency-domain (FDFD) complex-wavevector Bloch solver for the computation of band structures and modal properties of two-dimensional radiating periodic photonic devices. We first present the complex-wavector electromagnetic eigenvalue problem formulation and implementation of the solver with PML boundary conditions. We then analyze the modal properties of uniform and unidirectional, array-antenna-inspired, fiber-to-chip grating couplers [1,2] and low-loss, Bloch mode waveguide crossing arrays [3].

## 2. Formulation of the Complex-Wavevector Solver with Perfectly Matched Layers

Starting from the wave equation for the transverse electric field, $\Psi(x,y)$, of a periodic two-dimensional system and invoking Bloch's theorem to substitute $\Psi(x,y) = e^{jkx}\Phi(x,y)$, we derive the equivalent wave equation for the periodic Bloch amplitude over a unit cell of the structure, $\Phi(x,y)$, discretized on the Yee interleaved grid [10]

$$[\hat{\partial}_x\tilde{\partial}_x + \hat{\partial}_y\tilde{\partial}_y + jk(\hat{\partial}_x + \tilde{\partial}_x) - k^2 + k_0^2 n_{i,j}^2]\Phi_{i,j} = 0 \tag{1}$$

where $k_0$ is the free-space wavenumber, $k$ is the wavevector of the Bloch-Floquet wave in coordinate $x$, and $n(x,y) = n(x+a,y)$ is the refractive index distribution of the structure periodic in $x$. Eq. (1) has the form of a quadratic eigenvalue problem with eigenvalue $k$ which we recast into an equivalent linear eigenvalue problem [7]

$$\begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} \Phi \\ k\Phi \end{bmatrix} = k \begin{bmatrix} 0 & -C \\ I & 0 \end{bmatrix} \begin{bmatrix} \Phi \\ k\Phi \end{bmatrix} \tag{2}$$

where $I$ is the identity matrix and $A$, $B$, and $C$ are matrix operators derived from Eq. (1). To enable the simulation of radiating structures, we introduce PML boundary conditions in transverse coordinate $y$ utilizing complex-coordinate stretching

$$\tilde{y}(y) = y + jf(y) \tag{3}$$

where the modified complex $\tilde{y}$ coordinate is a combination of real coordinate $y$ and an imaginary function, with $f(y) \neq 0$ in the PML region. This change causes the radiated waves incident on the PML boundary to be transformed into exponentially decaying waves in the PML region allowing for absorption of leaky mode radiation instead of fictitious reflection of the radiation back into the solution space.

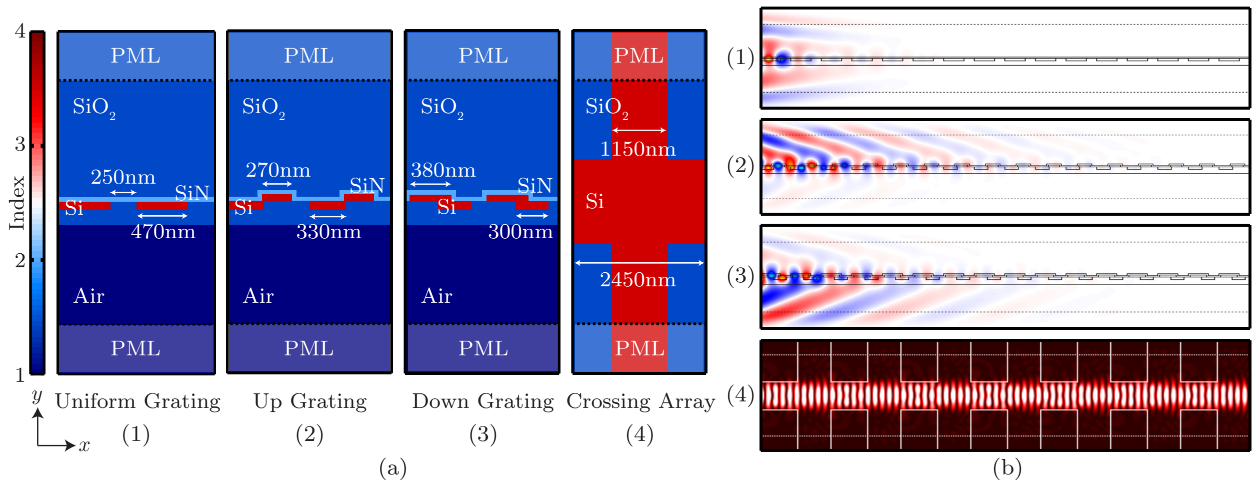


Fig. 1: (a) Two unit cells of (1) uniform, (2) upward, and (3) downward propagating grating couplers [1] and a unit cell of a (4) waveguide crossing array [4] showing index distributions and dimensions; (b) Bloch mode profiles at 1280 nm free-space wavelength of the gratings (1–3) and at 1200nm of the crossing array (4) simulated with PML boundaries in *y* and periodicity in *x*.

## 3. Numerical Results and Discussion

We use the proposed complex-wavevector solver to simulate the leaky-wave Bloch modes of periodic fiber-to-chip grating couplers, including a uniform design (1), as well as recently proposed upward (2) and downward (3) unidirectional radiation designs inspired by antenna array concepts [1,2], with structures shown in Figs. 1a(1–3). Figs. 1b(1–3) show the simulated Bloch modes of the three grating couplers demonstrating their radiation properties. For the uniform grating, each silicon tooth acts as a scatterer to collectively radiate the field in both the up and down directions as a non-uniform plane wave. In contrast, for the upward and downward unidirectional gratings, a two-element unit cell design with effective scattering centers separated in both *x* and *y* by a quarter wavelength leads to constructive interference in one direction and destructive in the other allowing for engineered unidirectional radiation. The complex-wavevector solver supports the formulation of rigorous synthesis algorithms for such sophisticated grating designs.

Next, we demonstrate, for the first time, rigorous computation of a low-loss, unidirectional, breathing Bloch mode proposed to exist at an imaginary anticrossing due to radiative coupling and used as the basis for ultra-low-loss waveguide crossing arrays [3]. Fig. 1a(4) gives the index distribution of a unit cell of the crossing array while Fig. 1b(4) shows the simulated field profile. Each crossing point causes slight scattering of the field, resulting in imaginary radiative splitting enabling low- and high-loss modes and confirming the existence of these Bloch modes.

## 4. Conclusions

The demonstrated complex-wavevector solver with PML boundary conditions is expected to enable powerful techniques for the rigorous synthesis of advanced periodic and quasi-periodic photonic devices including grating couplers, waveguide crossing arrays, and photonic crystals with future work focusing on synthesis of low-loss, non-uniform, adiabatic structures for efficient grating-based coupling. This work was supported by the University of Colorado Discovery Learning Apprenticeship undergraduate research program and NSF award number ECCS-1128709.

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